

Bell inequality for quNits with binary measurements

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Abstract

We present a generalized Bell inequality for two entangled quNits. On one quNit the choice is between two standard von Neumann measurements, whereas for the other quNit there are N^2 different binary measurements. These binary measurements are related to the intermediate states known from eavesdropping in quantum cryptography. The maximum violation by \sqrt{N} is reached for the maximally entangled state. Moreover, for $N = 2$ it coincides with the familiar CHSH-inequality.

1 Introduction

Recently there has been an increasing interest in generalizing results known for qubits to arbitrary dimensions. In this respect both quantum cryptography [1, 5] and various types of Bell inequalities [6], [7]-[11] have been generalized. Here we combine the two, in the sense that we present a Bell inequality for two quNits (N dimensional quantum systems), where the basic idea of the construction is inspired by quantum cryptography.

In the quantum cryptographic protocol, known as BB84 [12], the legitimate users Alice and Bob, both chose between measuring in one of the two mutually unbiased bases A and A' . However, an eavesdropper performing intercept/resend eavesdropping may chose to perform her measurements in what is known as the intermediate basis or the Breidbart basis [13]. In two dimensions it is possible to form two intermediate bases, but the eavesdropper needs only to make use of one of them. Turning to the Clauser-Horne-Shimony-Holt-inequality (CHSH) [14] for

two entangled qubits. The maximal violation is obtained when on the first qubit the measurement settings are A and A' , and on the second qubit the two intermediate bases.

It is this observation which lies at the heart of the construction of the inequality we present here. The intermediate states may be generalized to any dimension. However, in higher dimension the intermediate states do in general not form bases. But the projectors corresponding to these intermediate states can be used as binary measurements.

This idea leads to an inequality for two entangled quNits, where on the first quNit the choice of measurement is between two mutually unbiased bases A and A' , but on the second quNit the choice is between N^2 mutually incompatible binary measurements. These measurements correspond to the projectors of all the possible intermediate states. We find that the limit from a local variable point of view is 2, whereas the quantum mechanical limit is $2\sqrt{N}$.

It should be emphasized that, the inequality we present here differs in various aspects from the ones which have recently been presented in the literature. First of all the choice of measurements: Usually it is assumed that Alice and Bob each have two measurement settings. Here Alice has again two, but Bob has N^2 . Second, due to the special choice of measurements the construction of the inequality is easily generalized to any dimension. And finally, but very important, this inequality has contrary to other inequalities in higher dimensions, maximal violation for the maximally entangled state. This means that this inequality may be used as a measure of entanglement.

We show the full construction of the inequality for two qutrits and shortly discuss the generalization to arbitrary dimension. In section 2, we define the intermediate states for qutrits and in section 3 we obtain the corresponding inequality. In section 4 we show how to extend this result to any dimension. And then since the strength of a Bell inequality is often measured in terms of its resistance to noise, in section 5 we shortly discuss this issue for the inequality we have obtained. In section 6 we have the conclusions.

2 The intermediate states

In this and the next section we consider qutrits, i.e. $N = 3$.

The Bell inequality we are about to present is derived from the measurement settings, therefore we first define all the measurements involved. The measurement setting on one side, let's say the side of Alice,

correspond to two mutually unbiased basis A and A' and on the side of Bob, the measurement settings correspond to all the intermediate states which may be constructed from these two bases. Here the A -basis is chosen as the computational basis,

$$|a_0\rangle, |a_1\rangle, |a_2\rangle \quad (1)$$

where the states satisfy $\langle a_k | a_l \rangle = \delta_{kl}$. The A' -basis is chosen as the Fourier transform of the computational basis, i.e.

$$|a'_k\rangle = \frac{1}{\sqrt{3}} \sum_{n=0}^2 \exp\left(\frac{2\pi i kn}{3}\right) |a_n\rangle, \quad (2)$$

again these states satisfy

$$\langle a'_k | a'_l \rangle = \delta_{kl}, \quad \text{and moreover} \quad \langle a_k | a'_l \rangle = \frac{\exp(i\phi_{kl})}{\sqrt{(3)}}, \quad (3)$$

which means that the two bases are mutually unbiased, and that the distance between any states from the two different bases is $\cos(\theta) = 1/\sqrt{(3)}$.

The intermediate states are obtained by forming all possible pairs of states from the two bases. They are shown in the table below

| | a'_0 | a'_1 | a'_2 |
|-------|----------|----------|----------|
| a_0 | m_{00} | m_{01} | m_{02} |
| a_1 | m_{10} | m_{11} | m_{12} |
| a_2 | m_{20} | m_{21} | m_{22} |

where m_{ij} is understood as the intermediate state between the states $|a_i\rangle, |a'_j\rangle$, the first index always refers to the state from A and the second to the state from A' .

In quantum cryptography an eavesdropper, performing the simple intercept/resend eavesdropping strategy, may use the intermediate states to make a guess of the identity of each state send by Alice. Since the eavesdropper learns the basis in which the particle was originally prepared, she uses the intermediate states in the following way: suppose the eavesdropper in a measurement finds the state $|m_{ij}\rangle$, if she subsequently learns that the basis was A , she concludes that most probably the original state was $|a_i\rangle$, whereas if she learns that the basis was A' , she will guess that most probably the state was $|a'_j\rangle$. This means that she wants to optimize the conditional probability

$$p(m_{ij}|a_i) = p(m_{ij}|a'_j) = \text{max value.} \quad (4)$$

In other words she wants to optimize her probability for guessing the state correctly — independently of the basis. But at the same time she also wants the errors to be evenly distributed between the wrong states, i.e.

$$p(m_{ij}|a_k) = p(m_{ij}|a'_l) \quad k \neq i \text{ and } l \neq j. \quad (5)$$

The intermediate state, $|m_{ij}\rangle$, fulfill these requirements [15]. In terms of the two basis states $|a_i\rangle$ and $|a'_j\rangle$, it can be written as

$$|m_{ij}\rangle = \frac{1}{\sqrt{C}} \left(\exp(i\phi_{ij})|a_i\rangle + |a'_j\rangle \right) \quad (6)$$

where $C = 2(1 + 1/\sqrt{3})$ is the normalization constant, and the phase comes from the overlap between $|a_i\rangle$ and $|a'_j\rangle$. This leads to the conditional probability

$$p(m_{ij}|a_i) = p(m_{ij}|a'_j) = \frac{1 + \frac{1}{\sqrt{3}}}{2} = \frac{1}{2} + \frac{1}{2\sqrt{3}} \quad (7)$$

This can also be recognized as the cosine squared of half the angle, i.e. $\cos^2(\theta/2) = \frac{1+\cos(\theta)}{2}$. Which indeed shows that the intermediate state $|m_{ij}\rangle$, is as the name indicates, lying exactly between the states $|a_j\rangle$ and $|a'_j\rangle$. The probability for obtaining a wrong state is

$$p(m_{ij}|a_k) = p(m_{ij}|a'_l) = \frac{1}{2} \left(\frac{1}{2} - \frac{1}{2\sqrt{3}} \right) \quad (8)$$

In this way the total probability for making an error is $\frac{1}{2} - \frac{1}{2\sqrt{3}}$.

Notice that there has been made no requirement for orthogonality and indeed it may be checked that none of the nine states $|m_{ij}\rangle$ are orthogonal¹. However, each of the nine states can be associated with a projector $|m_{ij}\rangle\langle m_{ij}|$, which may be identified as a binary measurements. These nine mutually incompatible binary measurements are the measurement settings on Bob side.

3 The Bell inequality

Assume that the two observers Alice and Bob share many maximally entangled state of two qutrits. In the two bases A and A' this state may

¹However, it turns out that the nine states constitute a generalized measurement namely a so called POVM. We have, $\sum_{k,l=0}^2 \frac{1}{3} |m_{ij}\rangle\langle m_{ij}| = \mathbb{1}$.

be written as

$$\begin{aligned}
|\psi\rangle &= \frac{1}{\sqrt{3}}(|a_0, a_0\rangle + |a_1, a_1\rangle + |a_2, a_2\rangle) \\
&= \frac{1}{\sqrt{3}}(|a'_0, a'_0\rangle + |a'_1, a'_2\rangle + |a'_2, a'_1\rangle)
\end{aligned} \tag{9}$$

Notice that in the A' basis in order for the results, obtained by Alice and Bob, to be perfectly correlated does not mean that they will find the same state! For example, if Alice finds the state $|a'_2\rangle$, the state $|a'_1\rangle$ is the one which makes Bob perfectly correlated with Alice.

In order to write down the Bell inequality in a simple way, it is convenient to assign values to the various states, this assignment is shown in the table below

| value | A | A' | M_0 | M_1 | M_2 |
|-------|---------------|----------------|------------------|------------------|------------------|
| 0 | $ a_0\rangle$ | $ a'_0\rangle$ | $ m_{00}\rangle$ | $ m_{01}\rangle$ | $ m_{02}\rangle$ |
| 1 | $ a_1\rangle$ | $ a'_1\rangle$ | $ m_{11}\rangle$ | $ m_{12}\rangle$ | $ m_{10}\rangle$ |
| 2 | $ a_2\rangle$ | $ a'_2\rangle$ | $ m_{22}\rangle$ | $ m_{20}\rangle$ | $ m_{21}\rangle$ |

Notice that the $|m_{kl}\rangle$ -states have been organized in three sets, so that the value assigned to a given state is given by the first index. Moreover this organization into the sets M_0 , M_1 and M_2 , simplifies the notation in what follows. However, it is important to remember that the states in each set are **not** orthogonal, in other words they do not form three orthogonal bases.

Contrary to how Bell inequalities usually are presented, we here first present the quantum limit and only afterwards the local variable limit. The Bell inequality is obtained as the sum of probabilities for when the results of the measurements on the two qutrits are correlated and from this sum subtract all the probabilities for when the results are not correlated, i.e.

$$\begin{aligned}
B_3 &= \sum p(\text{results correlated}) \\
&\quad - \sum p(\text{results not correlated})
\end{aligned}$$

Now suppose that Alice measures in the A -basis and Bob measures a projector in the M_0 set. For this combination of measurements, there are the following contributions to the sum B_3 :

$$\begin{aligned}
P(M_0 = A) &= p(m_{00} \cap a_0) + p(m_{11} \cap a_1) + p(m_{22} \cap a_2) \\
&= \frac{1}{2} + \frac{1}{2\sqrt{3}}
\end{aligned} \tag{10}$$

$$\begin{aligned}
P(M_0 \neq A) &= p(m_{11} \cap a_0) + p(m_{22} \cap a_0) + p(m_{00} \cap a_1) + p(m_{22} \cap a_1) \\
&+ p(m_{00} \cap a_2) + p(m_{11} \cap a_2) \\
&= \frac{1}{2} - \frac{1}{2\sqrt{3}}
\end{aligned} \tag{11}$$

where $P(M_0 = A)$ should be read as: Bob measures a projector in M_0 and Alice measures A and Bob obtains the value which is correlated with Alice's result - hence the correct value. On the other hand $P(M_0 \neq A)$ means that Bob is not correlated with Alice, and hence obtain an error. The probability $p(m_{kl} \cap a_n) = p(m_{kl}|a_n)p(a_n)$ is the joint probability for obtaining both $|a_n\rangle$ and $|m_{kl}\rangle$.

The same is the case if Alice measures in A and Bob the projectors in the sets M_1 or M_2 , and again if Alice measures A' and Bob the projectors in M_0 . This gives the contribution, from the $M_1 - A$ combination of measurements: $P(M_1 = A) = p(m_{01} \cap a_0) + p(m_{12} \cap a_1) + p(m_{20} \cap a_2) = \frac{1}{2} + \frac{1}{2\sqrt{3}}$ and $P(M_1 \neq A) = \frac{1}{2} - \frac{1}{2\sqrt{3}}$. And from the $M_2 - A$ combination: $P(M_2 = A) = \frac{1}{2} + \frac{1}{2\sqrt{3}}$ and $P(M_2 \neq A) = \frac{1}{2} - \frac{1}{2\sqrt{3}}$. And finally from the $M_0 - A'$ combination: $P(M_0 = A') = \frac{1}{2} + \frac{1}{2\sqrt{3}}$ and $P(M_0 \neq A') = \frac{1}{2} - \frac{1}{2\sqrt{3}}$.

Consider now the case where Alice measures in A' and Bob measures the states in the set M_1 . In this case Bob consistently finds a value which is two higher (modulus 3) than the one which correlates him with Alice. To see this, assume for example that Bob has the state $|a'_0\rangle$ which is assigned the value 0. But the state which gives the correct identification of this state is $|m_{20}\rangle$, which is assigned the value 2. Similar for the other states, which leads to $P(M_1 = A' + 2) = p(m_{20} \cap a'_0) + p(m_{01} \cap a'_1) + p(m_{12} \cap a'_2) = \frac{1}{2} + \frac{1}{2\sqrt{3}}$ and $P(M_1 \neq A' + 2) = \frac{1}{2} - \frac{1}{2\sqrt{3}}$.

Whereas if Alice measures in A' and Bob the states in M_2 , he consistently finds a value which is 1 higher than the value which correlates him with Alice, i.e. $P(M_2 = A' + 1) = \frac{1}{2} + \frac{1}{2\sqrt{3}}$ and $P(M_2 \neq A' + 1) = \frac{1}{2} - \frac{1}{2\sqrt{3}}$.

The sum B_3 may now be written and evaluated:

$$\begin{aligned}
B_3 &= P(M_0 = A) - P(M_0 \neq A) \\
&+ P(M_1 = A) - P(M_1 \neq A) \\
&+ P(M_2 = A) - P(M_2 \neq A) \\
&+ P(M_0 = A') - P(M_0 \neq A') \\
&+ P(M_1 = A' + 2) - P(M_1 \neq A' + 2) \\
&+ P(M_2 = A' + 1) - P(M_2 \neq A' + 1) \\
&= 2\sqrt{3}
\end{aligned} \tag{12}$$

The quantity B_3 is a sum of joint probabilities and if written in full it consists of 54 terms. A local variable model which tries to attribute

definite values to the observables will reach a maximum value of 2. This may be checked numerically, but it can also be argued as is done in the following.

Since a_0 , a_1 and a_2 are measured simultaneously in a single measurement of the basis A , only one of them can come out true in local variable model. The same is the case for the a'_0 , a'_1 and a'_2 , which are measured as the basis A' . This means that, for example, if a_0 is true, meaning that a measurement of A will result in the outcome a_0 , then all probabilities involving a_1 and a_2 must be zero. It is different for the $|m_{ij}\rangle$ states since they are measured independently and hence they may all be true at the same time in a local variable model.

Now assume that according to a local variable model a_i and a'_j are true, at the same time in principle all the states m_{kl} could be true too. The question is now what will be the contributions from the various m -states. There are several possibilities. The state m_{kl} where both indices are different will only give negative contribution to the sum B_3 , since it fails to make the correct identification of any of the two basis states, and therefore only give rise to errors, i.e. contribute with -2 . Whereas a state m_{il} or m_{kj} , where one index is correct, will lead to a correct identification of one of the basis states, but since it fails to correctly identify the other, the net result is that these m -states give no contribution to the total sum.

The only state which will give a positive contribution to the total sum is m_{ij} which identifies correctly both a_i and a'_j , hence give a contribution of $+2$. From this it is seen that the maximum value according to local variables is 2, i.e.

$$B_3 \leq 2 \tag{13}$$

However, we have already seen that quantum mechanically it is possible to violate this limit. Quantum mechanically the limit is $2\sqrt{3}$.

It has been checked numerically that $2\sqrt{3}$ is indeed the maximal quantum mechanical limit for this sum of probabilities and that the quantum mechanical maximum is indeed reached for the maximally entangled state. Moreover it has also been shown, using "polytope software" [7, 16], that inequality (13) is optimal for the measurement settings which we have presented here.

4 Extension to arbitrary dimension N

In this section we show how to construct the inequality in any dimension N . We again assume that Alice and Bob share many maximally

entangled states.

Consider again the two bases A and A' , where the first is the computational basis and the second the Fourier transformed, each one now containing N basis vectors. Since the two bases are mutually unbiased, the distance between any state from one basis to any state in the other basis, is $\cos(\theta) = 1/\sqrt{N}$. The intermediate states may be constructed in exactly the same way as in three dimension, which means by forming all pairs of states from the two bases. Since the intermediate states (there are N^2) are defined as the ones lying exactly between a pair of states, which means that the distance from the intermediate state $|m_{ij}\rangle$ to the states $|a_i\rangle$ and $|a'_j\rangle$ is $\cos(\theta/2) = \sqrt{\frac{1}{2} + \frac{1}{2\sqrt{N}}}$. Which means that the probability of correct identification is

$$p(m_{ij}|a_i) = p(m_{ij}|a'_j) = \frac{1}{2} + \frac{1}{2\sqrt{N}} \quad (14)$$

and the probability of an error is

$$p(m_{ij}|a_k) = p(m_{ij}|a'_l) = \frac{1}{N-1} \left(\frac{1}{2} - \frac{1}{2\sqrt{N}} \right). \quad (15)$$

As in the case for two qutrits, it is convenient to assign values to the various states. In the table below is shown the values and the organization of the states into the sets M_0 - M_{N-1} :

| value | A | A' | M_0 | M_1 | \dots | M_{N-1} |
|---------|-------------------|--------------------|-----------------------|---------------------|---------|-----------------------|
| 0 | $ a_0\rangle$ | $ a'_0\rangle$ | $ m_{00}\rangle$ | $ m_{01}\rangle$ | \dots | $ m_{0,N-1}\rangle$ |
| 1 | $ a_1\rangle$ | $ a'_1\rangle$ | $ m_{11}\rangle$ | $ m_{12}\rangle$ | \dots | $ m_{10}\rangle$ |
| \dots | \dots | \dots | \dots | \dots | \dots | \dots |
| $N-1$ | $ a_{N-1}\rangle$ | $ a'_{N-1}\rangle$ | $ m_{N-1,N-1}\rangle$ | $ m_{N-1,0}\rangle$ | \dots | $ m_{N-1,N-2}\rangle$ |

Keeping the same notation as in the previous sections the Bell inequality for any dimension may be written

$$\begin{aligned}
B_N &= \sum_{i=0}^{N-1} P(M_i = A) - \sum_{i=0}^{N-1} P(M_i \neq A) \\
&+ \sum_{i=0}^{N-1} P(M_i = A' + N - i) - \sum_{i=0}^{N-1} P(M_i \neq A' + N - i) \\
&\leq 2
\end{aligned} \quad (16)$$

The local variable limit can again be argued as for the case of two qutrits, namely the only m -state which will give a positive contribution is the

one where both indices are the same as for the true basis states. Any other m -state will either give a negative contribution to the total sum or no contribution at all.

If it is written out in full it consists of $2N \times N^2$ terms. Since again the inequality is the sum of all the correct guesses, subtracting all the wrong guesses, the quantum mechanical limit is found to be

$$\text{QM} = 2N \left(\left(\frac{1}{2} + \frac{1}{2\sqrt{N}} \right) - \left(\frac{1}{2} - \frac{1}{2\sqrt{N}} \right) \right) = 2\sqrt{N} \quad (17)$$

Hence we have obtained an inequality where the violation increases with the square-root of the dimension.

It is important to realize that in the case $N = 2$ the inequality B_2 , is the famous CHSH-Inequality. In this case the two bases can be taken as the z -basis and the x -basis of a spin-1/2. If considering these two bases as axes on a great circle of the sphere, the intermediate states are the ones which lies at ± 45 degrees. Notice that in this case the two sets of intermediate states M_0 and M_1 actually do form two orthogonal bases.

5 Resistance to noise

In the resent papers on Bell inequalities, the strength of the inequality has been measured in terms of it's resistance to noise. The question is how much noise can be added to the maximally entangled state, $|\psi\rangle$, and still obtain a Bell violation. The more noise which can be added the better, since this means that the inequality is robust.

Until recently the noise considered was the uncolored noise, which means that the quantum state becomes

$$\rho_{mix} = \lambda_{mix} |\psi\rangle\langle\psi| + (1 - \lambda_{mix}) \frac{\mathbb{1}}{N} \quad (18)$$

The Bell inequality we have presented here reaches the classical limit, $B_N = 2$ for $\lambda_{mix}^{B_N} = \frac{N-1}{N+\sqrt{N}-2}$. For $N = 3$ this is $\lambda_{mix}^{B_3} = \frac{2}{1+\sqrt{3}} \simeq 0.73$. In comparison, the inequality presented by Collins, Gisin, Linden, Massar and Popescu (CGLMP) [11] is more robust to this kind of noise since they find a violation until $\lambda_{mix}^{CGLMP} \simeq 0.69$.

However it has recently been argued that the use of uncolored noise in this measure lead to problems [17, 18]. At the same time a new idea was introduces, namely instead of mixing the maximally entangled state with the maximally mixed state, to mix it with the closest separable state, i.e.

$$\rho_{cs} = \lambda_{sep} |\psi\rangle\langle\psi| + (1 - \lambda_{sep}) \rho_{sep} \quad (19)$$

where $\rho_{sep} = \frac{1}{N} \sum_{i=0}^{N-1} |a_i, a_i\rangle\langle a_i, a_i|$ [19]. Making use of ρ_{sep} leads to $\lambda_{sep}^{B_N} = \frac{N-\sqrt{N}}{N+\sqrt{N}-2}$, which for $N = 3$ is $\lambda_{sep}^{B_3} = \frac{3-\sqrt{3}}{1+\sqrt{3}} \simeq 0.46$. Whereas the CGLMP inequality again has $\lambda_{sep}^{CGLMP} \simeq 0.69$. Which means that the inequality we introduce here is much more resistant to this kind of noise.

It should however be stressed that the same measurement settings have been used in both evaluation of λ , and that the CGLMP inequality has been optimized to be resistant to the uncolored noise. It is nevertheless interesting to see how robustness of the B_N inequality change depending on the different noise added to the system.

6 Conclusion

We have presented a Bell inequality for quNits. The classical limit for this particular sum of joint probabilities is 2 - independent of the dimension. Whereas quantum mechanically it is possible to obtain a violation which increases with the square-root of the dimension, namely $2\sqrt{N}$. One of the interesting features of this inequality are the measurements which lead to the maximal violation. On Alice's side we have the usual two standard measurements of two mutually unbiased bases, but on Bob's side there is the choice of N^2 binary measurements. These measurements, which are represented by non-orthogonal projectors, correspond to the intermediate states of the two bases used by Alice. Intermediate states are known from intercept/resend eavesdropping in quantum cryptography.

It was the observation that for qubits, the intermediate states are both used in quantum cryptography and as the maximal settings for the CHSH-inequality which lead us to this construction of Bell inequalities in higher dimension. For $N = 2$, we therefore also recover the familiar CHSH-inequality.

This inequality further more has the advantage of being easily derived in any dimension. This is again due to the measurement settings. Since both the choice of measurements on Alice's side and on Bob's side are easily generalized to arbitrary dimension, so is the inequality.

Until recently the strenght of an inequality has been measured in terms of its resistance to uncolored noise. The inequality we present here is less resistant to this kind of noise than others. On the other hand it was recently argued [18] that the use of uncolored noise leads to problems. Instead it was suggested to mix the maximally entangled state with the closest separable state. Using this kind of noise we have shown that the inequality presented here is much more robust than the

CGLMP inequality.

Finally, it should also be mentioned that, in contrast to several other inequalities which have been presented recently, we have maximal violation for the maximally entangled state. This means that the inequality which we present here may be used as a measure of entanglement.

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